Calculating Risk Aversion using Sharpe Ratios

Jack Adeney

December 18, 2025

PROOF OF CONCEPT

There are many methods for calculating a portfolio manager's level of risk aversion based on their portfolio structure. One such measure involves computing Sharpe Ratio optimal portfolios and using this as a benchmark (Sharpe, 1966). Although I am not completely up-to-date with the behavioral finance literature, measuring the risk aversion of a manager can help firms predict behavioral biases of those generating portfolios and then adjust accordingly. The methodology outlined below provides a toy example for how somebody may be able to do this when observing simple portfolios.

Assume at first that there is a finite set of assets which can be indexed $i = 1, ..., n$. Each asset has an expected return, R_i, and an associated expected risk, σ_i. We let $\vec{R}$ represent the vector of expected returns and Σ the variance-covariance matrix. Although most markets allow the trade of significantly more complicated derivatives and instruments, we will assume that a portfolio is a convex combination of the underlying assets. Specifically, each portfolio, P, can be represented as a vector of weights $\vec{w}$ such that $\vec{w} \geq 0$ and $\vec{w} \cdot \vec{1} = 1$. This condition prevents decision makers from leveraging or shorting stocks. The expected return of a portfolio P with weights $\vec{w}_p$ is defined as

$$R_P = \vec{w}^T \vec{R}$$

Similarly, the expected risk associated with this portfolio is

$$\sigma_P = \sqrt{\vec{w}^T \Sigma \vec{w}}$$

Each of the feasible portfolios can be illustrated in the R^2 space where the x-axis represents the expected risk of the portfolio and the y-axis represents the expected return. *Figure 1* provides an example of two assets (red) plotted in the risk-return domain. The blue line is what is referred to as the 'efficient frontier, and represents the risk-return tradeoff of all possible convex combinations of the two assets.

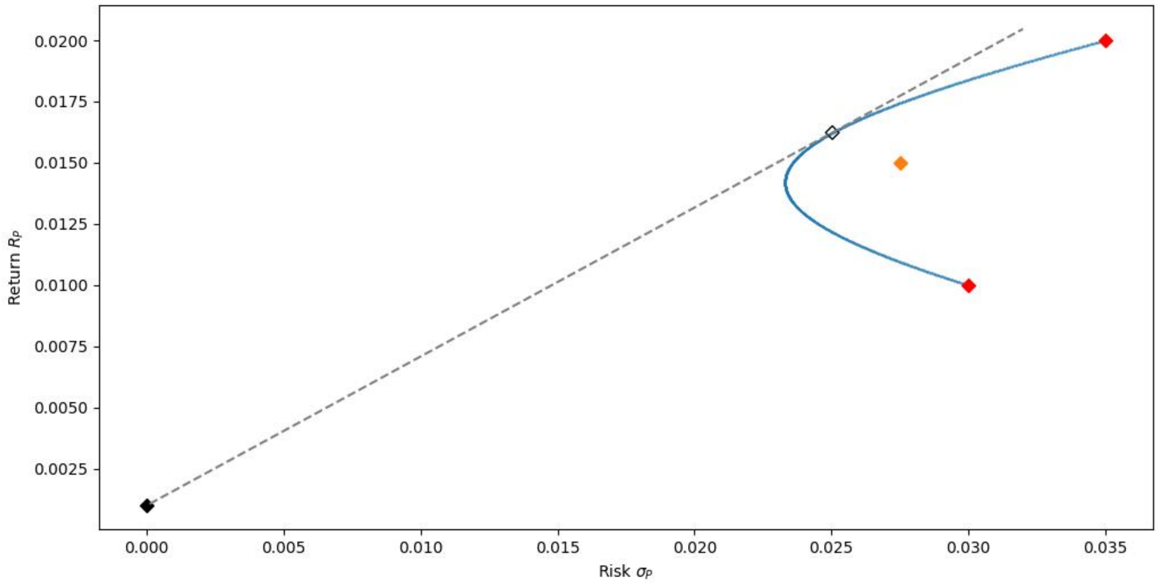

Figure 1: Efficient Frontier

We also assume that there is a risk-less asset with variance 0 and return R_f. The observability of this asset is fundamental for the construction of the Sharpe Ratios and is depicted by the black diamond in *Figure 1*.

The benefit of the Sharpe Ratio approach is that it provides a standardized measure for all portfolios constructed over the same set of underlying assets. The Sharpe Ratio of a portfolio P is defined as

$$SR_P := \frac{R_P - R_f}{\sigma_P}$$

and can be constructed for each of the portfolios. The sharpe-ratio explicitly represents the risk-return trade-off for each of the portfolios, and it seems natural that we would want to maximize return while minimizing risk. This is going to occur at the portfolio where the Sharpe Ratio is maximized. This portfolio, often referred to as the 'tangency portfolio' occurs at the point where the tangent of the efficient frontier runs through the risk free asset. It is depicted in *Figure 1* by the grey dashed line.

Now, notice that any portfolio along the dashed line in between the risk-less asset and the tangency portfolio can be constructed by creating a convex combination of the two. This is where we introduce our notion of risk aversion.

Typically, a highly risk-averse decision maker would not be willing to consume all the risk associated with the tangency portfolio. In which, case, they would place a sizeable weight on the risk-less asset as opposed to the portfolio. This implies that they will choose a point along that line close to the risk-less asset. A decision maker with very high tolerance to risk is likely to choose something very close to the tangency portfolio. If we were allowing for short sales, then the portfolio may even involve shorting the risk-less asset in order to get a portfolio along the line that provides more risk than the tangency portfolio.

There are many models of risk aversion that we could use as our standard. One of the most common from the finance literature is the Markowitz utility function, which is defined as follows;

$$U(P) = R_P - \frac{\lambda}{2}\sigma_P^2 \qquad \lambda \geq 0$$

As λ increases, the cost of risk increases, meaning that the decision maker is more likely to sacrifice expected return in exchange for reducing risk.

Now assume that the decision maker has chosen a portfolio somewhere within the feasible region. As an example, take the portfolio, P' where $R_{P'} = 0.015$ and $\sigma_{P'} = 0.0275$. This point is the orange diamond on *Figure 1*. The question becomes, can we find a measure of this decision maker's risk aversion based on their portfolio selection?

The methodology is as follows. First, find the point on the tangency line that is closest to P'. This point is going to correspond to a linear combination of the risk-less asset and the tangency portfolio. It is also going to be associated with an expected return and risk. Next, we can find the parameter λ that makes this portfolio the utility maximizing portfolio on the tangency line. Comparing these λ values for different portfolios allows us to compare attitudes towards risk.

Denote the portfolio on the tangency line closest to the portfolio P as $\hat{P}$. There is going to exist some w_f such that $R_{\hat{P}} = w_f R_f + (1 - w_f)R_{\hat{P}}$. The expected standard deviation associated with this portfolio will be $(1 - w_f)\sigma_{\hat{P}}$.

Take these values and substitute into the utility function:

$$\begin{aligned} U(\hat{P}) =& R_{\hat{P}} - \frac{\lambda}{2}\sigma^2_{\hat{P}} \\ =& w_f R_f + (1 - w_f)R_{\hat{P}} - \frac{\lambda}{2}(1 - w_f)^2\sigma^2_{\hat{P}} \end{aligned}$$

Differentiate with respect to w_f, set to zero, and solve for λ.

$$\begin{aligned} \frac{d}{dw_f}U(\hat{P}) :& R_f - R_{\hat{P}} + \lambda w_f \sigma^2_{\hat{P}} = 0 \\ \lambda =& \frac{R_{\hat{P}} - R_f}{w_f \sigma^2_{\hat{P}}} \end{aligned}$$

References

W. F. Sharpe. Mutual fund performance. *The Journal of business*, 39(1):119–138, 1966.